the fixed and scaled speedups of the parallel FDTD algorithm were provided, demonstrating the efficiency and the scalability of the algorithm on the highly parallel Intel Delta. The existing parallel FDTD algorithm is capable of supporting upwards of one billion degrees of freedom on the Intel Delta, and is capable of solving a problem of this magnitude in quite reasonable amounts of time. With the rate of advances of RISC processors and dynamic random access memory, high performance computers will be capable of handling 10's of billions of degrees of freedom in a fraction of the time using highly scalable algorithms such as the FDTD in the near future.

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# Scattering from a Circular Dielectric Post Embedded in a Grounded Dielectric Sheet Waveguide

E. Sawado, K. Ishibashi, and K. Hatakeyama

***Abstract*—A systematic method for obtaining the scattered electric field in a grounded dielectric sheet waveguide is presented. It is shown that point-matching method can be used for an explicit calculation of the integral equation to estimate the scattering from a circular dielectric post embedded in the grounded sheet. Magnitude of the reflection coefficient as a function of dielectric constant is given.**

## I. Introduction

In 1968 Schwinger published the lecture notes on the problem of electromagnetic scattering by a circular dielectric rod in a rectangular waveguide [1]. It was described that, for some special relations between the frequency, the dielectric constant, and the radius of the rod, the reflection coefficient becomes equal to zero [2], [3]. This problem has been a subject of interest to researchers for many years. In particular, attention was focused on the dip shown on the curve of reflection coefficient, illustrating as a function of the dielectric constant of the post. This phenomenon is due to volume resonance of the post.

The aim of this paper is to present a theory of scattering by a dielectric post (dielectric constant; $\epsilon_k = 2\epsilon_o \sim 500\epsilon_o$) embedded in a grounded dielectric sheet (dielectric constant $\epsilon_p = \kappa\epsilon_o$, the permittivity $\kappa = 2$). The point-matching method was used for the numerical estimation of magnitude of the reflection coefficient for the dielectric post with various permittivities. Fig. 1 shows the cross section of this structure. The structure is assumed to be uniform and infinite in both $x$ and $z$ directions. It is also assumed that substrate material is lossless.

For TE mode propagation, the electric field $E_y$ is a solution of

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} + k_0^2 E_y = 0 \qquad 0 < x \tag{1}$$

where $k_o^2 = \omega^2 \epsilon_o \mu_o$.

Mathematically the problem of relating a field to its source is that of integrating an inhomogeneous differential equation. Letting $j\omega\mu_o\delta(x-x')\delta(z-z')$ be the source function, we have the form

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} + \kappa k_o^2 E_y = -j\omega\mu_o\delta(x-x')\delta(z-z') \qquad -T < x < 0 \tag{2}$$

where $\kappa$ is the relative permittivity. We find the solution for $E_y$ by means of Laplace transform

$$g(x, \gamma) = \int_{-\infty}^{\infty} E(x, z) e^{\gamma z}\, dz. \tag{3}$$

Multiplying both side of (1) and (2) by $e^{\gamma z}$, and integrating from $-\infty$ to $+\infty$, we have

$$\frac{\partial^2 g}{\partial x^2} + (\gamma^2 + k_o^2) g = 0 \qquad 0 < x \tag{4}$$

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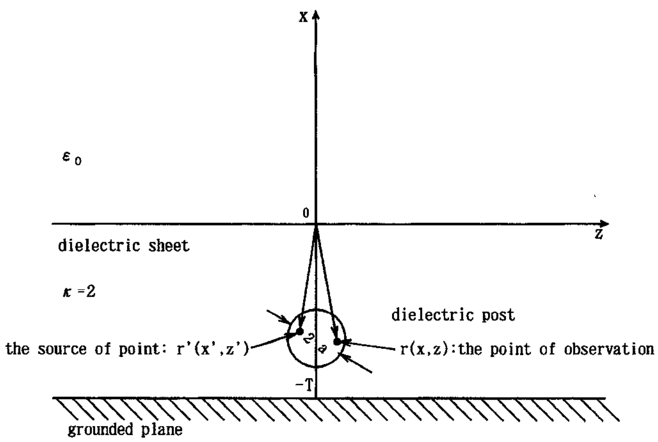


Fig. 1. Circular inductive post in dielectric sheet.

$$\frac{\partial^2 g}{\partial x^2} + (\gamma^2 + \kappa k_o^2) g = -j\omega\mu_o \delta(x - x') \quad -T < x < 0. \tag{5}$$

Equations (4) and (5) are satisfied by the functions of the form

$$g_1 = \frac{j\omega\mu_o e^{\gamma z'} \sin\{h(x'+T)\} e^{jhx'}(1+R)e^{-jlx}}{N(h, T, R, x')} \qquad 0 < x \tag{6}$$

$$g_2 = \frac{j\omega\mu_o e^{\gamma z'} \sin\{h(x'+T)\} e^{jhx'}(e^{-jhx} + \mathrm{R}e^{jhx})}{N(h, T, R, x')} \qquad x' < x < 0 \tag{7}$$

$$g_3 = \frac{j\omega\mu_o e^{\gamma z'}(1 + \mathrm{R}e^{2jhx'}) \sin\{h(x'+T)\}}{N(h, T, R, x')} \qquad -T < x < x' \tag{8}$$

where

$$N(h, T, R, x') = jh \sin\{h(x'+T)\}(1 - \mathrm{R}e^{2jhx'}) + h\cos\{h(x'+T)\}(1 + \mathrm{R}e^{2jhx'}) \tag{9}$$

$$h^2 = \gamma^2 + \kappa k_o^2 = \gamma^2 + k^2, \qquad l^2 = \gamma^2 + k_o^2 \tag{10}$$

and $T$ is the thickness of the sheet. Here $R$ is the reflection coefficient at an upper interface of the dielectric sheet given by

$$R = (h - l)/(h + l). \tag{11}$$

Inverting (3) gives the solution for $E_y$, which can be written in the form

$$E_y = G(r; r') = \frac{1}{2\pi} \int_c g(x, \gamma; r') e^{-\gamma z} \, d\gamma \tag{12}$$

where the contour $C$ is determined such that $G(r; r')$ behaves proper at infinity. $G(r; r')$ is interpreted as the diffracted electric field intensity $E_y$ of the point of observation ($r = x, z$) due to a unit current located at ($r' = x', z'$).

The branch point is situated at $\gamma$-plane, and is situated at $\gamma = +(\kappa)^{1/2} k_o$ on the upper half $\gamma$-plane, and the branch cut has been chosen to begin $(\kappa)^{1/2} k_o + jO$ to $(\kappa)^{1/2} k_o + j\infty$. The integration with respect to $\gamma$ is, by Cauchy's residue theorem, given by the sum of the integral along the path $-\infty$ to $+\infty$, the contribution from the integral about the branch cut $\int_{bc}$, and the integral along the semicircle at infinity $\int_{c1}$

$$\int_c g(x, \gamma; r') e^{-\gamma z} \, d\gamma = \int_{-\infty}^{\infty} g(x, \gamma; r') e^{-\gamma z} \, d\gamma + \int_{bc} g(x, \gamma; r') e^{-\gamma z} \, d\gamma + \int_{c1} g(x, \gamma; r') e^{-\gamma z} \, d\gamma = 2\pi j \Sigma \text{ Residues}. \tag{13}$$

The integral $\int_{c1}$ vanishes at infinity, and the integral contribution to the residues is zero because no poles exist on the upper half of the $\gamma$-plane

$$\int_{-\infty}^{\infty} g(x, \gamma; r') e^{-\gamma z} \, d\gamma + \int_{bc} g(x, \gamma; r') e^{-\gamma z} \, d\gamma = 0. \tag{14}$$

Converting the branch-cut integral of (14) into $h$-plane, we have [4]

$$\int_{bc} g(x, \gamma; r') e^{-\gamma z} \, d\gamma = \int_{-\infty}^{\infty} g(x, \gamma; r') e^{-\gamma z} \frac{\partial \gamma}{\partial h} \, dh. \tag{15}$$

In order to evaluate (15) for large $r$ by the saddle-point technique, it is convenient to change the variables of integration and as follows

$$\gamma = jk \sin\phi, \qquad h = k\cos\phi \tag{16}$$

where $k^2 = \kappa k_o^2$. The integration about the branch-cut which may be solved by means of a saddle-point technique. This procedure is accurate only when the size of post is small relative to wavelength in the medium. Equation (12) is given by the sum of the four Hankel-like functions as

$$E_y = G(r; r') = -I_1 - R(\phi_{p2}) I_2 + I_3 + R(\phi_{p4}) I_4 \tag{17}$$

where $\phi_{p2}$, $\phi_{p4}$ are defined by (24) and (28) for $x' < x < 0$, and (32) and (36) for $-T < x < x'$, respectively. And, here, the functions $I_i$ ($i = 1, 2, 3$, and 4) are given by

$$I_i = F(k, \phi_{pi}) \frac{e^{jf_i(k, \phi_{pi}, r, r') + j(\pi/4)}}{[f_i(k, \phi_{pi}, r, r')/2]^{1/2}}. \tag{18}$$

Here $F_i$ functions are given by

$$F_i(k, \phi_{pi}) = \frac{k\cos(\phi_{pi})}{\Delta(\phi_{pi})} \frac{\pi^{1/2}}{2} \tag{19}$$

where

$$\Delta(\phi_{pi}) = jh(\phi_{pi}) \sin\{h(\phi_{pi})(x'+T)\}(1 - \mathrm{R}e^{2jh(\phi_{pi})x'}) + h(\phi_{pi})\cos\{h(\phi_{pi})(x'+T)\}(1 + \mathrm{R}e^{2jh(\phi_{pi})x'}). \tag{20}$$

And $f_i$ functions are, for $x' < x < 0$

$$f_1(k, \phi_{p1}, r, r') = k\cos(\phi_{p1})\{(z - z')^2 + (2x' + T - x)^2\}/(2x' + T - x) \tag{21}$$

$$\tan(\phi_{p1}) = (z' - z)/(2x' + T - x) \tag{22}$$

$$f_2(k, \phi_{p2}, r, r') = k\cos(\phi_{p2})\{(z - z')^2 + (2x' + T + x)^2\}/(2x' + T + x) \tag{23}$$

$$\tan(\phi_{p2}) = (z' - z)/(2x' + T + x) \tag{24}$$

$$f_3(k, \phi_{p3}, r, r') = -k\cos(\phi_{p3})\{(z - z')^2 + (T + x)^2\}/(T + x) \tag{25}$$

$$\tan(\phi_{p3}) = -(z' - z)/(T + x) \tag{26}$$

$$f_4(k, \phi_{p4}, r, r') = k\cos(\phi_{p4})\{(z - z')^2 + (T - x)^2\}/(T - x) \tag{27}$$

$$\tan(\phi_{p4}) = -(z' - z)/(T - x) \tag{28}$$

and, for $-T < x < x'$

$$f_1(k, \phi_{p1}, r, r') = k\cos(\phi_{p1})\{(z - z')^2 + (x + T)^2\}/(x + T) \tag{29}$$

$$\tan(\phi_{p1}) = (z' - z)/(x + T) \tag{30}$$

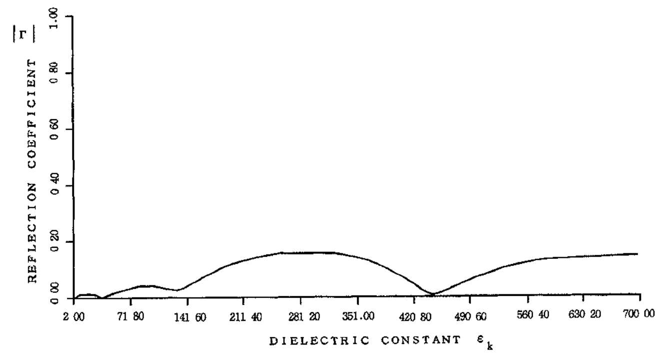


Fig. 2. Magnitude of the reflection coefficient as a function of dielectric constant ($\lambda_g = 3.12$ cm, $\kappa = 2$, $a = 2.2$ mm, $T = 16$ mm).

$$f_2(k, \phi_{p2}, r, r') = k\cos(\phi_{p2})\{(z-z')^2 + (2x'+T+x)^2\}/(2x'+T+x) \tag{31}$$

$$\tan(\phi_{p2}) = (z'-z)/(2x'+T+x) \tag{32}$$

$$f_3(k, \phi_{p3}, r, r') = -k\cos(\phi_{p3})\{(z-z')^2 + (x+T)^2\}/(x+T) \tag{33}$$

$$\tan(\phi_{p3}) = -(z'-z)/(x+T) \tag{34}$$

$$f_4(k, \phi_{p4}, r, r') = -k\cos(\phi_{p4})\{(z-z')^2 + (x+T-2x')^2\}/(x+T-2x') \tag{35}$$

$$\tan(\phi_{p4}) = -(z'-z)/(T+x-2x'). \tag{36}$$

The scattered electric field $E_y^s$ due to a whole current distribution in a dielectric post is, therefore

$$E_y^s(x, z) = -j\omega\mu_o \int_s \cdot G(x, z; x', z') J_y(x', z')\, dx'\, dz' \tag{37}$$

where $S'$ is the crosssection of the post. $G(x, z; x', z')$ of (17) is used for the estimation of the scattering integral given by (37). In the volume integral equation approach, we can replace the inductive dielectric post by equivalent volume currents $J_y$ given by [5]

$$J_y(x, z) = j\omega(\epsilon_p - \epsilon_k) E_y(x, z) \tag{38}$$

where

$$E_y(x, z) = E_y^i(x, z) + E_y^s(x, z). \tag{39}$$

Letting $E_y^i$ denote the incident field, and using (38) and (39), we can write the following integral equation for the unknown current distributions $J_y(x, z)$

$$E_y^i(x, z) = \frac{J_y(x, z)}{j\omega(\epsilon_p - \epsilon_k)} - E_y^s(x, z) \tag{40}$$

where the scattered electric field $E_y^s(x, z)$ is given by (37).

As a simple example of the above procedure we computed the reflection coefficient for a circular dielectric post of diameter a located in a dielectric sheet of thickness $T$ with its axis parallel to the transverse electric fields. To solve this problem, a point-matching method which is a direct moment method with the weighting functions being Dirac's delta functions, is used to obtain the scattered fields $E_y^s$. Magnitude of the reflection coefficient $|\Gamma|$ is given by

$$|\Gamma| = \left|\frac{E_y^s}{E_y^i}\right|_{\substack{z \sim -100\lambda_g \\ x = -T/2}}. \tag{41}$$

The cross sectional area $s'$ is divided into 360 small rectangular cells. The relevant parameters are $a = 2.2$ mm, $\lambda_0 = 3.12$ cm, and $\kappa = 2$. Fig. 2 shows the reflection coefficient as a function of the dielectric constant. This numerical estimation shows the resonant conditions of $\epsilon_p = 37.36\epsilon_o, 122.21\epsilon_o, 447.46\epsilon_o$.

## II. Conclusion

It is emphasized that this analysis is apparently powerful since it is not limited to the case of circular post but is general in that post of arbitrary crosssection, location, and number can be handled effectively.

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